

UR-1577
 ER/40685/936
 hep-ph/9907359
 July 1999

Lepton - Chargino Mixing and R-Parity Violating SUSY¹

Mike Bisset

Department of Physics, Tsinghua University, Beijing, 100084, China

Otto C.W. Kong, Cosmin Macesanu and Lynne H. Orr

Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627

Abstract. We present a study of charged lepton mass matrix diagonalization in R-parity violating SUSY. The case in which the bilinear couplings μ_i have large values is given special attention.

INTRODUCTION

R-parity violating SUSY is a subject which has enjoyed a lot of interest in the past few years. The possibility of mixing between particles and superpartners, allowed in these models, makes for very interesting phenomenology. However, due to the large number of parameters present, this subject is also quite difficult to study. Recently, it has been found (see [1]) that by working in a specific basis (single VEV parametrization) the analysis of the fermionic sector is greatly simplified, without loss of generality. In this basis, the only non-MSSM parameters that play a role in the leptonic phenomenology at tree level are the three RPV bilinear couplings μ_i .

The phenomenological consequences of this model in the fermionic sector have been extensively studied [2]. This paper aims to detail the technical aspects concerning the chargino-lepton mass matrix diagonalization. It has been found that for large values of the couplings μ_i (of order of hundreds GeV and above) this problem is not trivial. We should point out that our interest in this issue *here* is mostly theoretical: most of the range of μ_i values relevant to the discussion below is not

¹⁾ Presented by C. Macesanu at 21th Annual MRST (Montreal-Rochester-Syracuse-Toronto) Meeting on High-Energy Physics, Ottawa, 10-12 May 1999.

allowed by the experimental constraints (except in special cases; for more details on this subject see [2]).

The framework is the same as in [1]. The mass matrix can be written:

$$M_c = \begin{bmatrix} M_2 & g_v & 0 & 0 & 0 \\ g'_v & \mu_0 & 0 & 0 & 0 \\ 0 & \mu_1 & \bar{m}_1 & 0 & 0 \\ 0 & \mu_2 & 0 & \bar{m}_2 & 0 \\ 0 & \mu_3 & 0 & 0 & \bar{m}_3 \end{bmatrix}$$

where $g_v = g_2 v_u / \sqrt{2} = \sqrt{2} M_w \sin \beta$, $g'_v = g_2 v_d / \sqrt{2} = \sqrt{2} M_w \cos \beta$. Here, by \bar{m}_i we denote the Yukawa masses of the three leptons e, μ, τ . The physical masses will be denoted by m_i .

The aim is to go from the weak interaction fields, in terms of which the lagrangian is written initially, to the mass eigenstate (physical) fields, which can be observed experimentally. To this purpose, we rotate the left fields by a matrix U^L and the right fields by a matrix U^R . These rotations will also diagonalize the squared mass matrices:

$$U^{L\dagger} M^L U^L = U^{R\dagger} M^R U^R = \text{diag}\{M_{\chi_1}^2, M_{\chi_2}^2, m_i^2\}$$

where $M^L = M_c^\dagger M_c$, $M^R = M_c M_c^\dagger$.

The mixing between leptons and charginos naturally leads to changes in the couplings of these particles to the gauge bosons (see [3]). In the case of Z coupling we will have :

$$\begin{aligned} A_{ij}^L &= \tilde{A}_{ij}^L + (1 - 2\sin^2 \theta_W) \delta_{ij}, & \tilde{A}_{ij}^L &= U_{i1}^L U_{j1}^L \\ A_{ij}^R &= \tilde{A}_{ij}^R - 2\sin^2 \theta_W \delta_{ij}, & \tilde{A}_{ij}^R &= 2U_{i1}^R U_{j1}^R + U_{i2}^R U_{j2}^R \end{aligned}$$

The diagonal (δ_{ij}) terms are the SM values, while the \tilde{A} terms are consequences of the mixing. Being nondiagonal, they lead to anomalous Z couplings and non-standard decays (e.g. $Z \rightarrow e\mu$, $\mu \rightarrow eee$). It turns out that, in most cases, the anomalous left coupling is the important one; the anomalous right coupling is proportional with the product $m_i m_j$ and very small numerically. In what follows we will concentrate on the left rotation matrix.

I ANALYTIC DIAGONALIZATION OF MASS MATRIX

In this section we will analyze approximate analytical solutions to our diagonalization problem. The perturbative solution for small μ 's has been given in [1]; we present it here for comparison with further results:

$$U_{i1}^L = \mu_i \frac{\sqrt{2} M_w \cos \beta}{\Delta} , \quad i = e, \mu, \tau \quad (1)$$

$\Delta = \mu_0 M_2 - g_v g'_v$. This formula reveals to us one important fact: the strength of the mixing decreases at large $\tan\beta$. The phenomenological consequences of this behaviour have been analyzed in [1], [2].

The fact that the above formula doesn't work at large μ is apparent; indeed, as we increase μ_i , the components U_{i1}^L , as given by the above formulae, increase indefinitely, which is not allowed by U matrix unitarity.

It is possible to diagonalize the matrix without assuming that the μ_i are small. Instead, we will take the Yukawa masses to be small; actually, we will take them to be zero in the matrix M^L . Then, we get the following solution:

$$U^L = \begin{bmatrix} x_1 & x'_1 & \mu_1 \frac{g'_v}{\Delta_e} & \mu_2 \frac{g'_v \Delta}{\Delta_\mu \Delta_e} & \mu_2 \frac{g'_v \Delta}{\Delta_\tau \Delta_\mu} \\ x_2 & x'_2 & -\mu_1 \frac{M_2}{\Delta_e} & -\mu_2 \frac{M_2 \Delta}{\Delta_\mu \Delta_e} & -\mu_2 \frac{M_2 \Delta}{\Delta_\tau \Delta_\mu} \\ x \frac{\mu_1}{\mu_5} & x' \frac{\mu_1}{\mu_5} & \frac{\Delta}{\Delta_e} & -\frac{\mu_2 \mu_1 \alpha_2}{\Delta_\mu \Delta_e} & -\frac{\mu_3 \mu_1 \alpha_2}{\Delta_\tau \Delta_\mu} \\ x \frac{\mu_2}{\mu_5} & x' \frac{\mu_2}{\mu_5} & 0 & \frac{\Delta_e}{\Delta_\mu} & -\frac{\mu_3 \mu_2 \alpha_2}{\Delta_\tau \Delta_\mu} \\ x \frac{\mu_3}{\mu_5} & x' \frac{\mu_3}{\mu_5} & 0 & 0 & \frac{\Delta_\mu}{\Delta_\tau} \end{bmatrix} \quad (2)$$

where $\Delta_e^2 = \Delta^2 + \alpha_2 \mu_1^2$, $\Delta_\mu^2 = \Delta^2 + \alpha_2 (\mu_1^2 + \mu_2^2)$, $\Delta_\tau^2 = \Delta^2 + \alpha_2 (\mu_1^2 + \mu_2^2 + \mu_3^2)$, and $\alpha_2 = M_2^2 + g'^2_v$. The first two columns correspond to the two charginos, while the last three columns correspond to the leptons (e, μ, τ respectively).

Note that, in the approximation used, the physical masses of the particles (eigenvalues of the matrix M^L) are also zero; as a consequence, the three lepton eigenvectors are degenerate. To get the correct combination, we employ a limiting procedure: start from the exact eigenvectors, and let the electron mass, muon mass, and tau mass go to zero, in this order. If the lepton mass hierarchy holds also for the Yukawa masses ($\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$) it can be shown that this way we get the correct result (2).

Besides lepton eigenvectors, other quantities of interest obtained through diagonalization of the mass matrix are chargino masses. In the small Yukawa mass approximation, we get

$$M_{\chi_{1,2}} = \frac{\alpha_1 + \alpha_2}{2} \pm \sqrt{(\alpha_1 - \alpha_2)^2 + 4(M_2 g_v + \mu_0 g'_v)^2}$$

with $\alpha_2 = \mu_0^2 + g_v^2 + \mu_5^2$ ($\mu_5^2 = \mu_1^2 + \mu_2^2 + \mu_3^2$). Interpretation of the quantities α_1 and α_2 is straightforward; at large μ_5 , the mass of the heavier chargino is $M_{\chi_1} \approx \sqrt{\alpha_1} \approx \mu_5$, while the mass of the lighter chargino is $M_{\chi_2} \approx \sqrt{\alpha_2}$. Actually, it can be shown that the lighter chargino mass increases monotonically from the MSSM value (for $\mu_5 = 0$) to $\sqrt{\alpha_2}$ (for $\mu_5 \rightarrow \infty$). This behaviour has important phenomenological consequences. Consider the fact that the lower limit on the lighter chargino mass $M_{\chi_2} > 90$ GeV eliminates part of the (M_2, μ_0) plane in the MSSM. With R-parity violating terms, you can expect that this excluded region will shrink; indeed, if we make μ_5 big enough, it might potentially go away completely. The fact that M_{χ_2} is limited above by $\sqrt{\alpha_2}$ means that some region in the (M_2, μ_0) plane does in fact

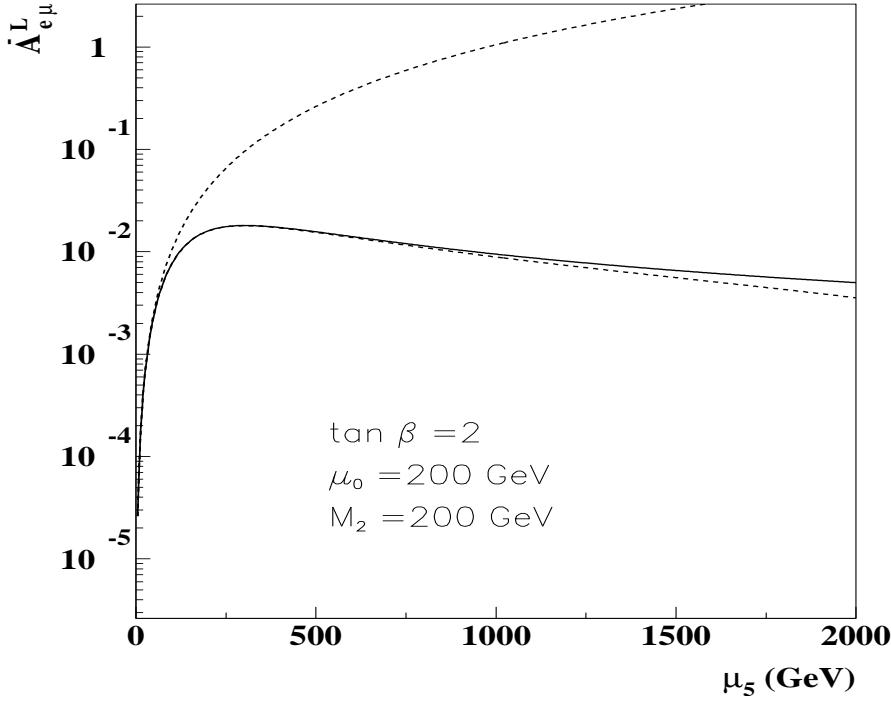


FIGURE 1. Strength of left-handed $Ze\mu$ anomalous coupling : dotted line: small μ approximation (1); solid line: small Yukawa mass approximation (2); dashed line: exact numerical result; μ_i ratio 1:1:1

remain excluded, no matter how strong the R-parity is violated. This region is given by the equation:

$$\sqrt{\alpha_2} < 90 \text{ GeV}$$

or, at large $\tan\beta$, $M_2 < 90$ GeV. This result is supported by the exact numerical analysis presented in [1], [2].

Let's turn back to the lepton eigenvectors and consider the anomalous lepton-lepton-Z couplings. For simplicity, let's take the left-handed $Ze\mu$ coupling:

$$\tilde{A}_{e\mu}^L = \mu_1 \mu_2 \frac{g_v'^2 \Delta}{\Delta_e^2 \Delta_\mu} \quad (3)$$

At small μ 's, the strength of the coupling increases with $\mu_1 \mu_2$, while at large μ 's it decreases like $1/\sqrt{\mu_5^2}$ (see Fig. 1). In between it will reach a maximum value:

$$\tilde{A}_{e\mu \ max}^L = C \frac{g_v'^2}{M_2^2 + g_v'^2}$$

C being a constant which depends only on the ratio μ_1/μ_2 . Experimental constraints on anomalous branching ratios or lepton number violating decays (which

can generally be written in the form $\tilde{A}_{e\mu}^L < A_0$) will then be satisfied not only in the region of small μ_5 , but also for large values of μ_5 . Moreover, if in some region of parameter space $\tilde{A}_{e\mu}^L \max < A_0$ for some process, then that particular process won't contribute at all to constraints on μ_5 values.

The analytic solution (2) derived in this section is not valid at arbitrarily large μ 's. The approach used to derive the lepton eigenvectors works only as long as $\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$. Once we get close to the boundaries of the region where the diagonalization problem has solutions (see next section) this relation does not hold anymore. However, the numerical results for the anomalous couplings \tilde{A}_{ij}^L show an even steeper decrease in this region than that given by (3). As a consequence, for sufficiently large μ_5 , constraints from electroweak processes like lepton or Z decay disappear. This region of large μ_5 is excluded on the basis of strong interaction processes (π decay, or neutrinoless beta decay). However, the corresponding case involving mainly large μ_3 may be phenomenologically viable [2].

II YUKAWA MASSES

To be able to perform an exact diagonalization of the mass matrix, we first have to find the Yukawa masses of the leptons. Note that this is not the standard problem, in which we have a matrix and we have to find eigenvalues. In this case, we know three of the eigenvalues (the physical masses m_i of the leptons) and we have to find some elements of the matrix itself (the Yukawa masses \bar{m}_i).

The direct approach would be to use the eigenvalue equations:

$$\det(M - \lambda I) = 0 \quad , \quad \lambda = m_e^2, m_\mu^2, m_\tau^2 \quad (4)$$

This is a three by three system of nonlinear equations, and is not easy to solve even numerically (except in particular cases). So, we will use another approach.

This approach is based on the observation that if we know the solution for some values μ_i^2 , we can find the solution at an neighbouring point $\mu_i^2 + d\mu_i^2$. For simplicity, let's consider the ratio of the μ 's fixed, and their magnitude given by a parameter t :

$$\mu_i = r_i \sqrt{t}$$

Write the eigenvalue equations:

$$E(t, \bar{m}_i^2, \lambda) = \det(M - \lambda I) = 0$$

To an infinitesimal modification in t will correspond an infinitesimal modification in the Yukawa parameters \bar{m}_i :

$$\frac{\partial E}{\partial t} + \frac{\partial E}{\partial \bar{m}_i^2} \frac{d\bar{m}_i^2}{dt} = 0 \quad (5)$$

Now, we can numerically integrate this system of linear equations from zero to whatever value of t we need.

What about the existence of solutions for our problem? Let's suppose we can solve the system (4); in order that the result make sense, we require the solutions to be real. This will restrict the allowable values of μ_i to some domain D in μ_i space. What this means, in terms of solutions derived with the help of (5), is that we can increase t only as long as we stay inside this domain. When we reach its boundary, usually what happens is that the determinant of the system (5) becomes zero, and we cannot solve for $d\bar{m}_i$.

Another relevant question is if this domain is simply-connected; that is, starting from $\mu_i = 0$, can we reach any point of it with a path formed by connected straight lines? In other words, does integrating the system (5) allow us access to all the solutions to (4)?

We do not know the answer to this question for the general case of three leptons. But, if we consider the simpler case of only two leptons (presume one of the μ_i is zero), we can write (4) in the form:

$$\begin{cases} a_1\bar{m}_1^2 + a_2\bar{m}_2^2 = s \\ a_1a_2\bar{m}_1^2\bar{m}_2^2 = p \end{cases}$$

with solution:

$$(a\bar{m}^2)_{1,2} = \frac{s}{2} \mp \sqrt{s^2 - 4p}$$

The quantity $A = s^2 - 4p$ becomes 0 for some value $\mu_5 = \mu_{5 \text{ max}}$ (which gives us the boundary for the domain D), and it can be shown that only for $\mu_5 < \mu_{5 \text{ max}}$ is A positive (necessary condition for real solutions). Note that $\mu_{5 \text{ max}}$ is generally around a few TeV.

Another interesting issue is the problem of lepton mass hierarchy in this model. In the Standard Model (or MSSM) we have $\bar{m}_e \ll \bar{m}_\mu \ll \bar{m}_\tau$. These relations need not hold in our R-parity violating scenario. Take, for example, the two lepton mixing case presented above. If $\mu_1 = \mu_2$, then, at $\mu_5 = \mu_{5 \text{ max}}$, we have $\bar{m}_1 = \bar{m}_2$ (this will happen at quite large μ_5 values, though; $\mu_{5 \text{ max}} \sim m_2/m_1$). If $\mu_1 > \mu_2$, it is even possible to get \bar{m}_1 greater than \bar{m}_2 . The next question is if this behaviour holds for the general case of three lepton mixing. The possibility of finding points in parameter space where the three Yukawa masses are of the same order of magnitude (or maybe even equal) is particularly interesting. Unfortunately, the technical difficulties encountered in working with the nonlinear system (4) have stopped us from getting an answer to this question so far.

III CONCLUSIONS

For large R-parity violating terms, the mixing between charginos and charged leptons has different characteristics than at small μ_i . We have presented approximate

analytical expressions for both regimes, which can help to understand numerical results.

The problem of existence of solutions of the system (4) - finding the Yukawa masses so that three of the mass matrix eigenvalues will be equal to the physical lepton masses - is still unsolved for the general case of three lepton mixing (although within the phenomenologically viable region, numerical solutions are always successfully obtained in [2]). For two lepton mixing it can be solved, and it has been shown that there are regions in parameter space where the Yukawa masses of the two particles are of the same order of magnitude.

REFERENCES

1. M. Bisset, O.C.W. Kong, C. Macesanu, L.H. Orr, Phys. Lett. **B430**:274-280, 1998
(hep-ph/9804282)
2. M. Bisset, O.C.W. Kong, C. Macesanu, L.H. Orr, (hep-ph/9811498)
3. M. Novakowsky, A. Pilaftsis, Nucl. Phys. **B461**:19-49, 1996